Supplementary Materials

Counterexample search in diagram-based geometric reasoning

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Supplementary Materials

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Experiment 1

Table S1

The 18 reasoning problems of Experiment 1 with the possible conclusions provided for each problem. For valid problems, when a label is written next to the conclusion, this means that the inference corresponds to one of the inference rules in the formal system of Avigad et al. (2009), in which case the label is the one of the rule in Avigad et al. (2009).

Valid Problems	Invalid Problems
1 Point A is **inside** circle α Point A is **on** line L ——————————— Line L **intersects** circle α (E-IR3) Line L **does not intersect** circle α	**2** Point A is **outside** circle α Point A is **on** line L ——————————— Line L **intersects** circle α Line L **does not intersect** circle α
3 Point A is **inside** circle α Circle α is **inside** circle β ——————————— Point A is **inside** circle β Point A is **on** circle β Point A is **outside** circle β	**4** Point A is **inside** circle α Circle β is **inside** circle α ——————————— Point A is **inside** circle β Point A is **on** circle β Point A is **outside** circle β
5 Line L **intersects** circle α Circle α **inside** circle β ——————————— Line L **intersects** circle β Line L **does not intersect** circle β	**6** Line L **intersects** circle α Line L **intersects** circle β ——————————— Circle α **intersects** circle β Circle α **does not intersect** circle β
7 Points A and B are **on the same side** of line L Points A and C are **on the same side** of line L ——————————— Points B and C are **on the same side** of line L (E-SSA4) Points B and C are **on opposite sides** of line L	**8** Points A and B are **on opposite sides** of line L Point C is **off** line L ——————————— Points A and C are **on the same side** of line L Points A and C are **on opposite sides** of line L

Table S1 Continued

9

Points *A* and *B* are **on opposite sides** of line *L*

Point *A* is **on** line *M*

Point *B* is **on** line *M*

Line *L* **intersects** line *M* (E-IR1)

Line *L* **does not intersect** line *M*

10

Points *A* and *B* are **on opposite sides** of line *L*

Point *A* is **off** line *M*

Point *B* is **off** line *M*

Points *A* and *B* are **on the same side** of line *M*

Points *A* and *B* are **on opposite sides** of line *M*

11

Point *A* is **on** circle α

Point *B* is **inside** circle α

Point *A* is **inside** circle β

Point *B* is **on** circle β

Circle α **intersects** circle β (E-IR5)

Circle α **does not intersect** circle β

12

Point *A* is **on** circle α

Point *B* is **inside** circle α

Point *A* is **inside** circle β

Point *B* is **inside** circle β

Circle β **intersects** circle α

Circle β **does not intersect** circle α

13

Point *B* is **between** points *A* and *C* **on** line *L*

Point *B* is **between** points *A* and *D* **on** line *L*

Point *B* is **between** points *C* and *D* **on** line *L*

Point *B* is **not between** points *C* and *D* **on** line *L* (E-BA7)

14

Point *B* is **between** points *A* and *C* **on** line *L*

Point *D* is **between** points *A* and *C* **on** line *L*

Point *B* is **between** points *A* and *D* **on** line *L*

Point *B* is **not between** points *A* and *D* **on** line *L*

15

Point *A* is **inside** circle α

Point *C* is **outside** circle α

Point *C* is **between** points *A* and *B* **on** line *L*

Point *B* is **inside** circle α

Point *B* is **on** circle α

Point *B* is **outside** circle α (E-CA3)

16

Points *A* and *B* are **on opposite sides** of line *L*

Point *C* is **inside** circle α

Point *C* is **on** line *L*

Point *A* is **inside** circle α

Point *A* is **on** circle α

Point *A* is **outside** circle α

17

Point *A* is **inside** circle α

Circle α is **inside** circle β

Point *B* is **on** circle β

Points *A* and *B* are **on opposite sides** of line *L*

Circle β **intersects** line *L*

Circle β **does not intersect** line *L*

18

Circle α **intersects** line *L*

Point *A* is **inside** circle α

Circle β **intersects** circle α

Point *B* is **inside** circle β

Points *A* and *B* are **on the same side** of line *L*

Points *A* and *B* are **on opposite sides** of line *L*
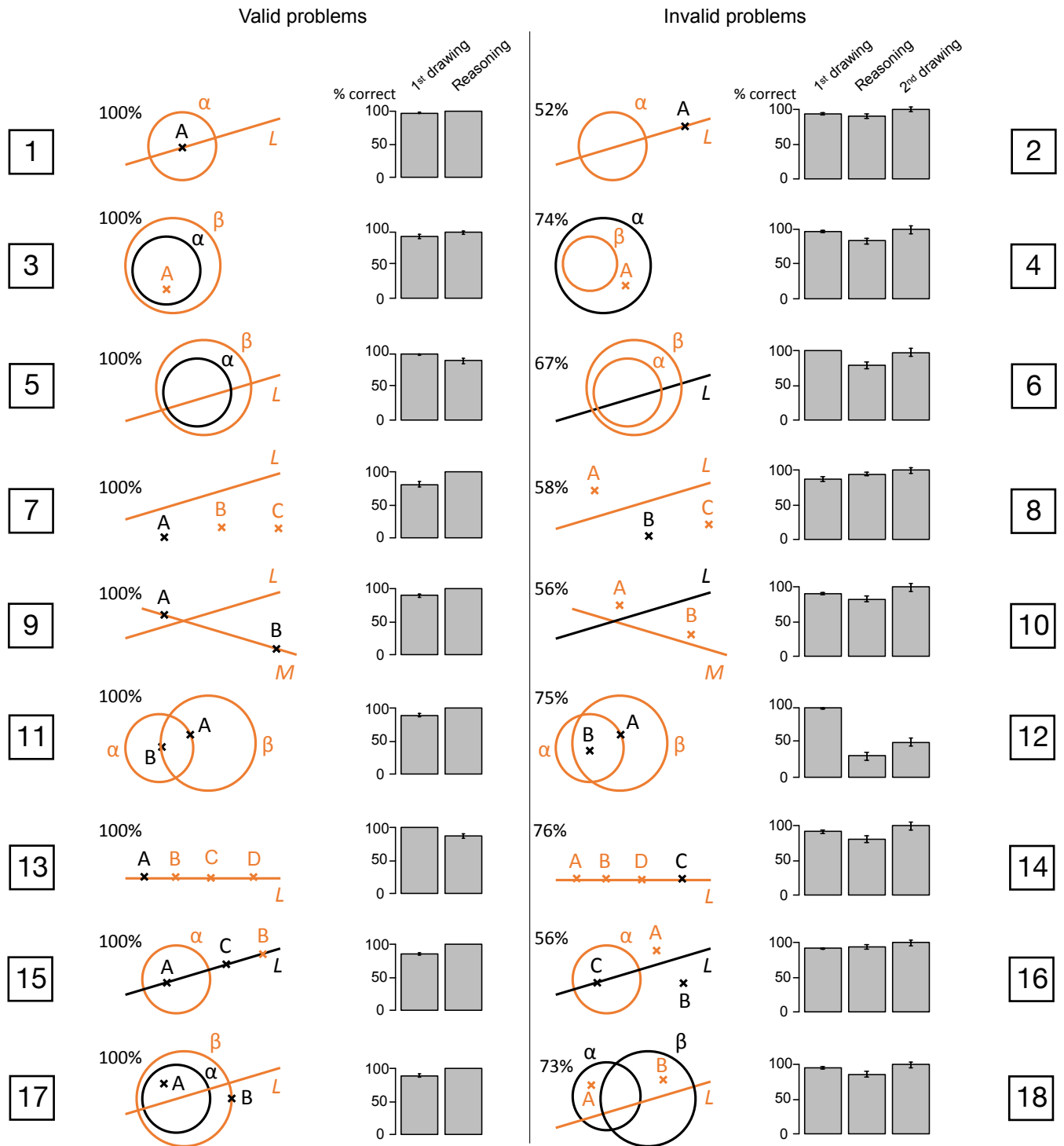
Table S2

Preferred topological configurations for the first drawing and percentages of correct first drawing, correct response to the reasoning question given a correct first drawing, and correct second drawing given a correct response to the reasoning question and a correct first drawing, for the 18 reasoning problems of Experiment 1. The objects in orange are those present in the conclusions. The valid (resp. invalid) problems are numbered from top to bottom with increasing odd (resp. even) numbers.

Experiment 2

Table S3

The 12 valid scanning problems of Experiment 2. The object in red is the variable object.
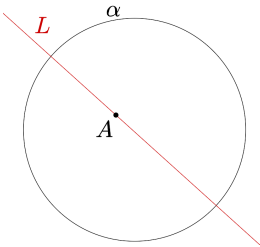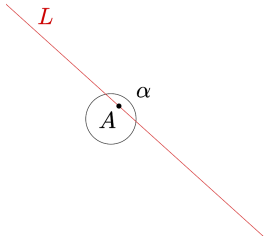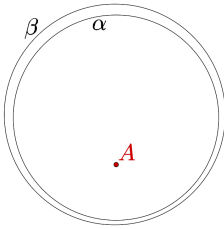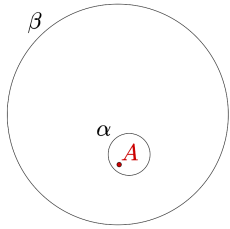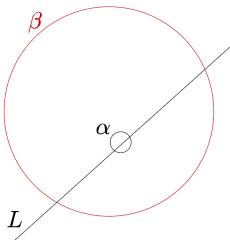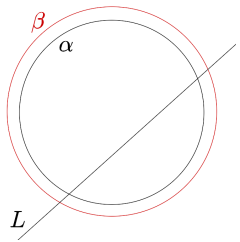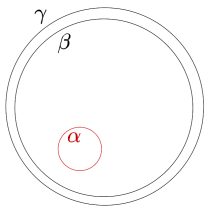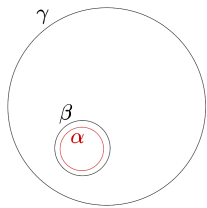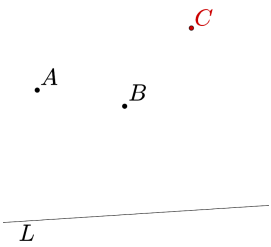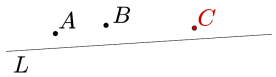
Constraint(s) and Conclusion	Image 1	Image 2
Point A is **on** line L Line L **intersects** circle α		
Point A is **inside** circle α Point A is **inside** circle β		
Circle α is **inside** circle β Line L **intersects** circle β		
Circle α is **inside** circle β Circle α is **inside** circle γ		
Points A and C are **on the same side of** line L Points B and C are **on the same side of** line L		

Table S3 Continued
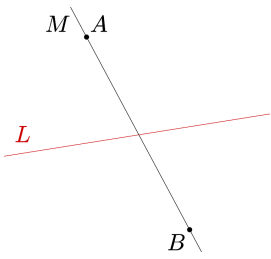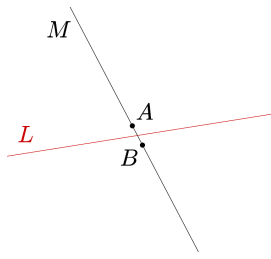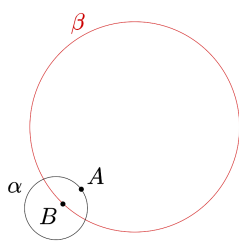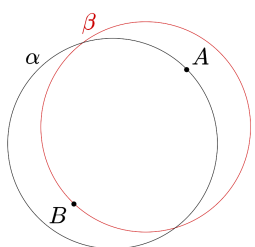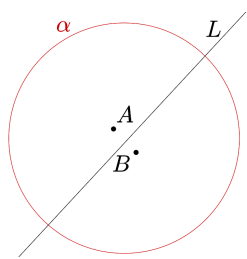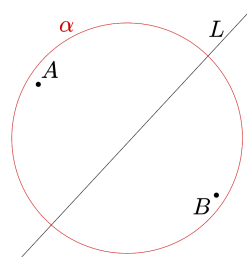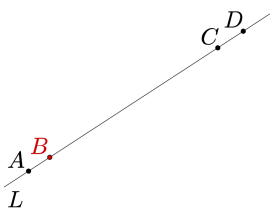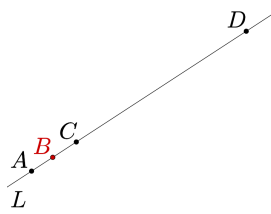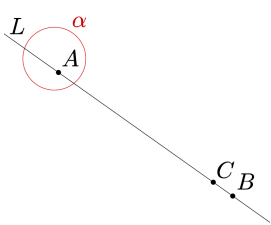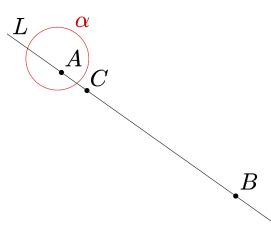
Points *A* and *B* are **on opposite sides of** line *L* Line *L* **intersects** line *M*	
Point *A* is **inside** circle *β* Point *B* is **on** circle *β* Circle *α* **intersects** circle *β*	
Point *A* is **inside** circle *α* Point *B* is **inside** circle *α* Circle *α* **intersects** line *L*	
Point *B* is **between** points *A* and *C* on line *L* Point *B* is **between** points *A* and *D* on line *L* Point *B* is **not between** points *C* and *D* on line *L*	
Point *A* is **inside** circle *α* Point *C* is **outside** circle *α* Point *B* is **outside** circle *α*	

Table S3 Continued

	High Counterexample	Low Counterexample
Points A and B are **on opposite sides** of line L <hr> Line L **intersects** circle β		
Points A and C are **on the same side of** line L <hr> Point A and B are on the same side of line L		

Table S4

The 12 invalid scanning problems of Experiment 2 with the images in the two conditions: with a framework of high counterexample density and with a framework of low counterexample density. The object in red is the variable object.

Constraint(s) and Conclusion	High Counterexample Density	Low Counterexample Density
Point A is **on** line L <hr> Line L **intersects** circle α		
Point A is **inside** circle α <hr> Point A is **outside** circle β		

Table S4 Continued
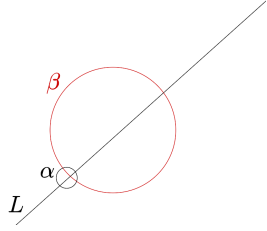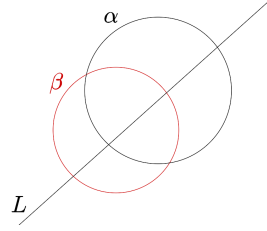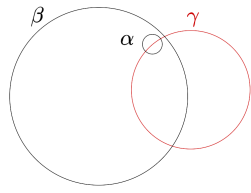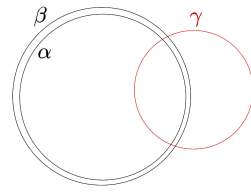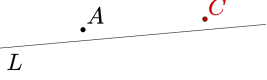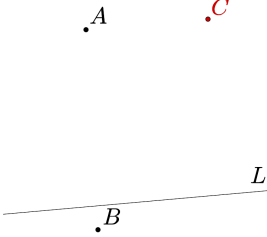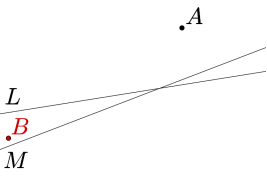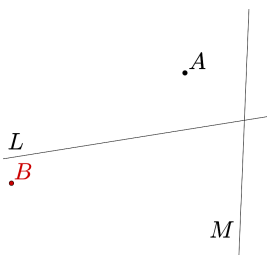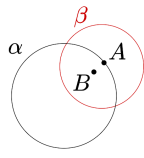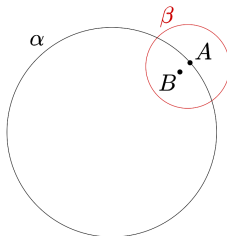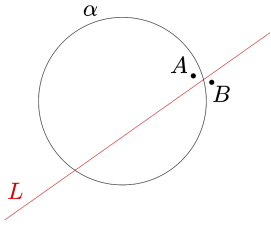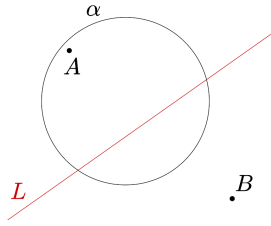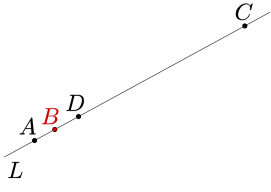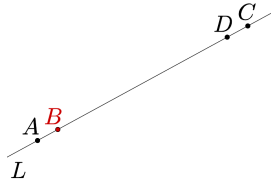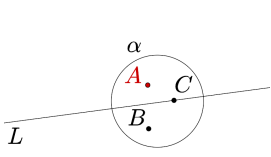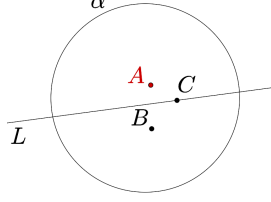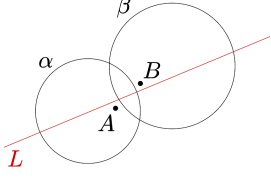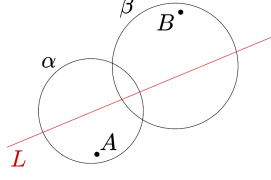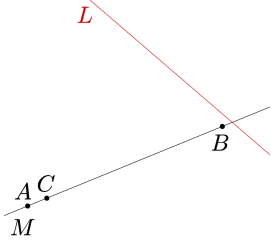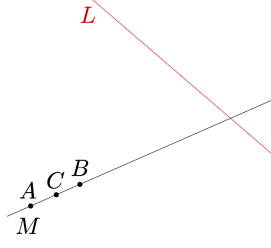
Line *L* **intersects** circle *β*

Circle *α* **intersects** circle *β*

Circle *β* **intersects** circle *γ*

Circle *α* **intersects** circle *γ*

Point *C* is **not on** line *L*

Points *A* and *C* are **on the same side of** line *L*

Points *A* and *B* are **on opposite sides of** line *L*

Point *B* is **not on** line *M*

Points *A* and *B* are **on the same side of** line *M*

Point *A* is **inside** circle *β*

Point *B* is **inside** circle *β*

Circle *β* **intersects** circle *α*

Table S4 Continued

Point A is **not on** line L Point B is **not on** line L Line L **intersects** circle α --- Points A and B are **on opposite sides of** line L	
Point B is **between** points A and C on line L --- Point B is **between** points A and D on line L	
Points A and B are **on opposite sides of** line L --- Point A is **inside** circle α	
Circle α **intersects** line L --- Points A and B are **on opposite sides of** line L	
Line L **intersects** line M Points A and C are **on the same side of** line L Points A and B are **on the same side of** line L	

Experiment 3

Table S5

The 12 valid scanning problems of Experiment 3. The object in red is the variable object.
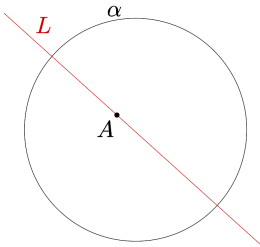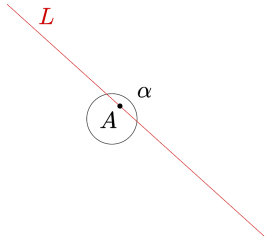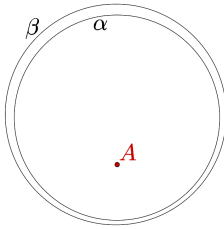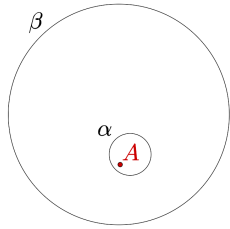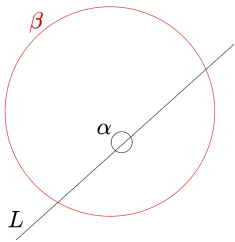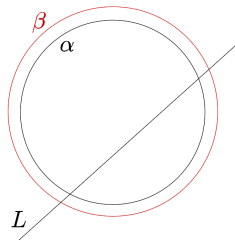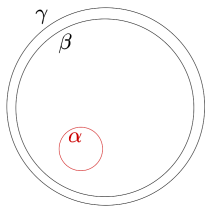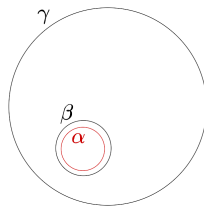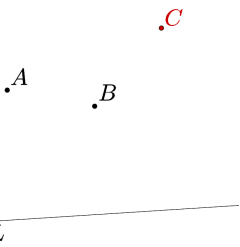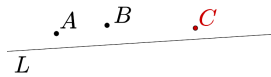
Constraint(s) and Conclusion	Image 1	Image 2
Point *A* is **on** line *L* Line *L* **intersects** circle *α*		
Point *A* is **inside** circle *α* Point *A* is **inside** circle *β*		
Circle *α* is **inside** circle *β* Line *L* **intersects** circle *β*		
Circle *α* is **inside** circle *β* Circle *α* is **inside** circle *γ*		
Points *A* and *C* are **on the same side of** line *L* Points *B* and *C* are **on the same side of** line *L*		

Table S5 Continued
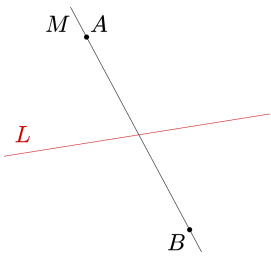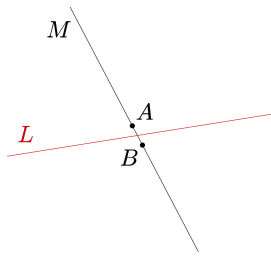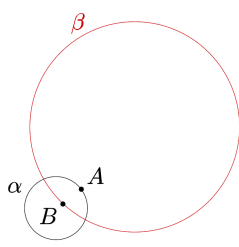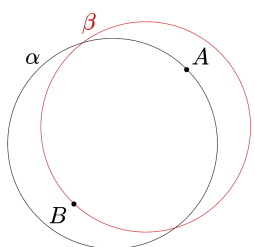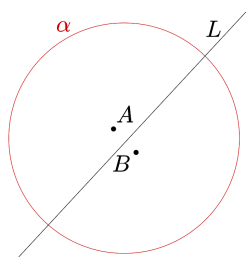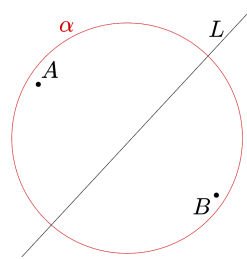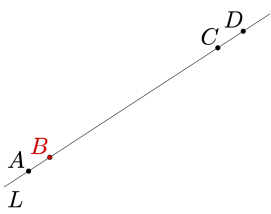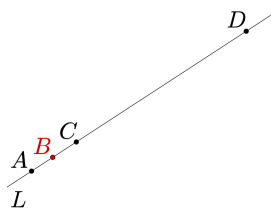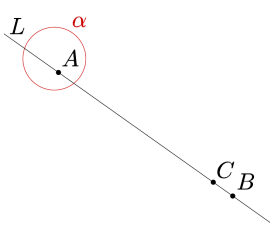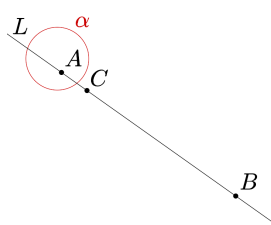
Points *A* and *B* are **on opposite sides of** line *L*

Line *L* **intersects** line *M*

Point *A* is **inside** circle β

Point *B* is **on** circle β

Circle α **intersects** circle β

Point *A* is **inside** circle α

Point *B* is **inside** circle α

Circle α **intersects** line *L*

Point *B* is **between** points *A* and *C* on line *L*

Point *B* is **between** points *A* and *D* on line *L*

Point *B* is **not between** points *C* and *D* on line *L*

Point *A* is **inside** circle α

Point *C* is **outside** circle α

Point *B* is **outside** circle α

Table S5 Continued
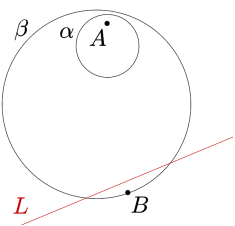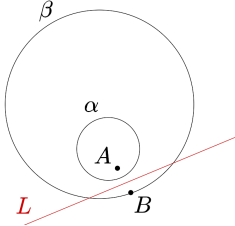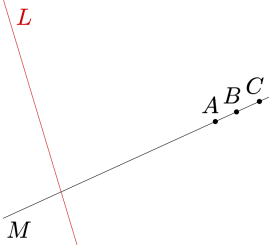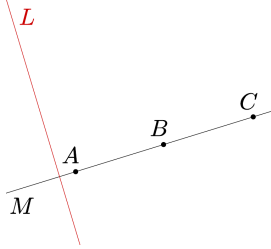
Points *A* and *B* are **on opposite sides** of line *L*	
Line *L* **intersects** circle *β*	
Points *A* and *C* are **on the same side of** line *L*	
Point *A* and *B* are on the same side of line *L*	

Table S6

The 12 invalid scanning problems of Experiment 3 with the images in the two conditions: with the variable object either close to or far from the closest counterexample(s). The object in red is the variable object.
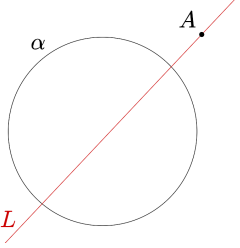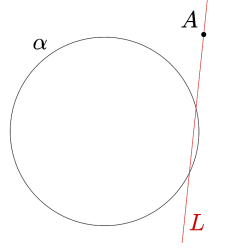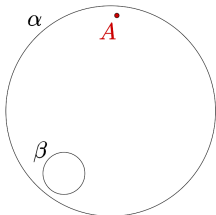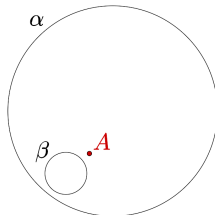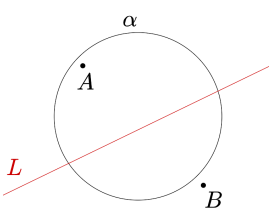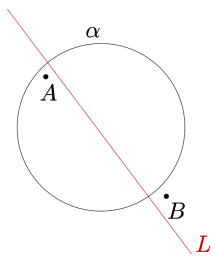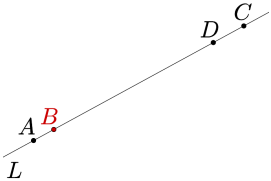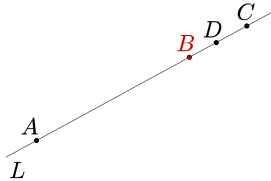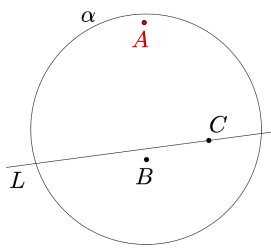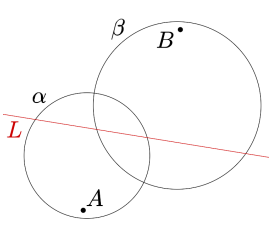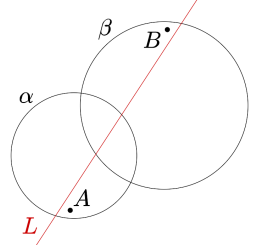
Constraint(s) and Conclusion	Far from the Closest Counterexample(s)	Close from the Closest Counterexample(s)
Point *A* is **on** line *L* Line *L* **intersects** circle *α*		
Point *A* is **inside** circle *α* Point *A* is **outside** circle *β*		

Table S6 Continued

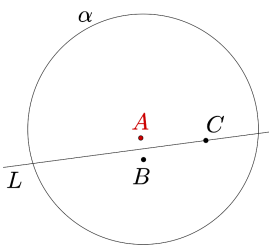|	
Line *L* **intersects** circle *β* Circle *α* **intersects** circle *β*	
Circle *β* **intersects** circle *γ* Circle *α* **intersects** circle *γ*	
Point *C* is **not on** line *L* Points *A* and *C* are **on the same side of** line *L*	
Points *A* and *B* are **on opposite sides of** line *L* Point *B* is **not on** line *M* Points *A* and *B* are **on the same side of** line *M*	
Point *A* is **inside** circle *β* Point *B* is **inside** circle *β* Circle *β* **intersects** circle *α*	

Table S6 Continued

Point *A* is **not on** line *L* Point *B* is **not on** line *L* Line *L* **intersects** circle α Points *A* and *B* are **on opposite sides of** line *L*	
Point *B* is **between** points *A* and *C* on line *L* Point *B* is **between** points *A* and *D* on line *L*	
Points *A* and *B* are **on opposite sides of** line *L* Point *A* is **inside** circle α	
Circle α **intersects** line *L* Points *A* and *B* are **on opposite sides of** line *L*	
Line *L* **intersects** line *M* Points *A* and *C* are **on the same side of** line *L* Points *A* and *B* are **on the same side of** line *L*	